Investigating engineering students' mathematical modelling competency

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ABSTRACT: In this article the author presents, illustrates, tests and refines a framework developed by Galbraith, Stillman, Brown and Edwards for identifying student blockages whilst undertaking modelling tasks during transitions in the modelling process [1]. This study investigates engineering students' modelling processes in one modelling activity. The data include information collected on individual students' and group written responses to the mathematical modelling activity, video-taped group discussions and classroom observation by the researcher. The data showed that engineering students have difficulty in the transition between different modes of mathematical representation, and the classifications of variables/parameters as known or unknown, implicit or explicit, independent or dependent variables are necessary.

INTRODUCTION

School mathematics instruction has created an impression among students of mathematics as being tedious, abstract and unrelated to the real world. Increasing numbers of researchers in mathematics education are concerned about the negative perceptions of students toward mathematics. They suggest the use of real-world problems in mathematics classes to link the world of mathematics to reality. Teachers and researchers, such as Lesh and Doerr [2] and Burkhardt [3], agree that mathematical modelling is an important aspect of mathematics education. Over the past ten years, it has become increasingly important to apply mathematics to other subjects, including engineering, nanotechnology, economics and biology. Many educators and researchers in mathematics education believe that this should be reflected in the classroom via mathematical modelling activities. Students should be availed with tools in addition to school mathematics and allowed to glimpse real-world mathematics outside the classroom.

The use of models and modelling in enhancing the instruction and learning of mathematics is an indispensable means of cultivating students' mathematical literacy, which they need in the new era of technology [2][4]. Several factors, such as entrance examinations and existing teaching materials fail to create a favourable environment for mathematical modelling in the current mathematics education situation in Taiwan. However, college students have less academic pressure and calculus is a fundamental course in college-level mathematics and engineering education. Students need to understand the concepts of calculus and be able to apply them. For engineering students, calculus is not only a specialised subject, but also knowledge that they will need in their future workplaces. Thus, integrating modelling activities into calculus courses is an appropriate approach in implementing mathematical modelling instruction.

This study presents mathematical modelling activities based on models and modelling perspectives and embedded into calculus courses to develop students' mathematical modelling competency. Teaching experiments in this study used the island approach proposed by Blum and Niss to integrate model-based teaching activities into formal activities for teaching calculus, and is used to avoid resistance from students who are used to traditional teaching [5]. The ultimate purpose of the teaching experiments is to foster students' modelling competency through a modelling process. By implementing such teaching experiments, the mathematical modelling process and competency of first year engineering students were investigated, which can be used as a reference for designing activities for teaching mathematical modelling to college students.

THEORETICAL BACKGROUND

Modelling is the process of establishing a model through understanding, analysing and exploring phenomena. During the modelling process, individuals are required to find answers to problems. More importantly, they may experience conceptual understanding, attempt representational information processing, and interpret the relationship between

models and phenomena, gradually building up the competency that they need. According to the procedures for developing models, a sound modelling process must include model-eliciting, model-exploring and model-adapting activities [2]. Model-eliciting activities are mainly used for stimulating students to think of a variety of ideas; model-exploring activities particularly emphasise the introduction of mathematical structures; model-adapting activities focus on integration and applications.

Researchers such as Kaiser [6] and Maaß [7] proposed six stages of the modelling cycle from a cognitive perspective. The modelling cycle begins with a real-world situation. After understanding the situation, individuals will construct their mental representation of it, then idealise, filter, simplify or structure the information in the situation to obtain a real model and transform it into a mathematical model through mathematisation. Finally, individuals implement mathematical treatments on this mathematical model to get a mathematical result and, then, interpret it as a real result and validate it. The validation result may show that the real result fails to meet requirements or that other aspects must be considered; thus, individuals need to restart the whole modelling process.

Mathematical modelling instruction aims to support students in learning mathematics. Through modelling, mathematics can be used to describe, understand and predict real-world situations. Hence, mathematical modelling can help students gain external mathematical experience and create the connections between mathematical concepts involved in modelling activities. Mathematical modelling involves multiple processes such as mathematisation, interpretation, communication and even application [2][7].

Unlike traditional problem solving, which focuses only on the representation of mathematical problems and solutions, mathematical modelling focuses on converting and interpreting contextual information, identifying potential problems, establishing models, and reinterpreting the premise, hypothesis, and possible errors of mathematical solutions. These processes are normally described in the form of stages. By following these processes, students can constantly refine and develop their mathematical models in a circular manner. Moreover, students need to be able to engage in mental activities when moving from one stage to another during the modelling process.

Competency indicates that individuals are able to make relevant decisions and implement proper actions in a real-world situation. These decisions and actions are essential for individuals in successfully handling real situations. As Blum and Leiß indicated, if teaching and learning are emphasised simultaneously, an individual-oriented *perspective* on *problem solving* is necessary to improve understanding of what students do when solving modelling problems, and to provide a better foundation for the diagnosis and involvement of educators [8]. This study adopts the modelling cycle proposed by Galbraith and Stillman as a research framework to investigate the mathematical modelling process and mathematical competency of first year engineering students [9].

METHODOLOGY

This study adopts an *interpretative orientation based on anti-positivism*, regarding case studies as a research strategy for closely examining the modelling process of students [10]. The mathematical modelling problem of this study is as follows:

A company is carrying out a cost-cutting exercise and requires help with an investigation into how it can reduce its transport costs. The company employs a number of drivers who drive a substantial distance every day. There has recently been a large increase in fuel costs and drivers can achieve a higher rate of miles per gallon from their vehicles by driving at a lower speed. This, however, increases journey times and the cost of the driver's time.

The data reported in this study were gathered from three calculus classes. The entire process of each class, which lasted for 70 minutes, was recorded and videotaped. The subjects in this study consisted of 54 first year university engineering students. These students were divided into ten groups comprising five to six students. The researcher observed and videotaped three groups: Groups A, D and F. After each class, the researcher had a retrospective interview with these three groups to understand their real intentions. The researcher showed a certain group of students a video regarding their situation in class and asked them to explain their behaviour in detail by asking questions, such as *How did you propose your ideas at this point?* This article reported the modelling process of the six students in Group D. The grades of these students in calculus were approximately average. Sam, as an instructor in the teaching experiments, had 15 years' experience teaching calculus in a university. He was willing to participate in this study because of his interest in fostering students' mathematical thinking through modelling.

In the first class, Sam first posed a mathematical problem and students had class discussions and worked on their own. After some students posed their questions, others could express their opinions. Sam guided or instructed students to become reflexive by asking students several questions such as *Why?*, *When?*, and *How will you do it?*. Then, students had group discussions and tried to understand, structure and simplify a real-world situation, and convert it into a problem statement. During this process, students gradually discovered and verified several keywords in the problem statement and were encouraged to convert a real-world problem into a problem statement based on keywords. In the second class, students needed to simplify or structure a problem situation based on a problem statement and to generate other real models. Students first held group discussions for 30 minutes. They were encouraged to re-examine a problem

statement, and asked to create variables, parameters, constants and symbolic representations based on keywords. In the second stage, Groups A, D, H and G were invited to share their reports and held class discussions. During the discussions, Sam occasionally asked students questions such as *Why did you think in this way?* and *Do you think this is the best way?* These questions provided students with material for discussion and enabled students to perceive the importance of examining arguments. Sam also took this opportunity to explain the similarities and differences between variables, parameters and constants. In the third class, students first used known mathematical knowledge to solve mathematical models on their own and, then, held group discussions to interpret mathematical solutions as real results.

RESULTS

Figure 1 shows the mathematical modelling competencies in each transition (cognitive activity) that were identified in this implementation of the task. Each element has two parts where key (generic) categories in the transitions between phases of the modelling cycle are indicated (in regular type) and illustrated (in capitals) with reference to the task. Evidence for selected examples of these activities is presented in the analysis of transitions that follows.

Real-world situations → Real-world models. In the first stage, students verified certain keywords in a problem statement, including the driving speed, costs in terms of diesel fuel and driver salary and transportation costs. There was a mathematical observation during the inquiry process. Inquiry and mathematical observation allowed students to surpass their preconceived opinions of real-world situations, especially when students had talks with others in group discussions. For example, *The truck travels at a constant speed, ignoring traffic lights and jams* was the important concept that was simplified in group discussions.

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1. *Real- world situations → Real- world models* (understand, simplify and interpret context)

1.1 Clarify the context of problems (DRIVE THE OPTIMAL DRIVING SPEED FOR A TRUCK TO MINIMISE TRANSPORTATION COSTS UNDER CONSIDERATIONS OF THE COST OF DIESEL FUEL AND THE DIRVER'S SALARY)
1.2 Simplify hypotheses (A TRUCK TRAVELS AT A CONSTANT SPEED, IGNORING TRAFFIC LIGHTS AND JAMS)

2. *Real - world models → Mathematical models* (CONSTRUCT HYPOTHESIS AND MATHEMATISATION)

2.1 Verify variables and parameters (THE DISTANCE DRIVEN BY A TRUCK IS A PARAMETER)
2.2 Use graphical representation (THE RELATIONSHIP BETWEEN THE SPEED OF A TRUCK AND THE NUMBER OF km/L OF DIESEL FUEL THE TRUCK CAN GET)
2.3 Use situational elements of graphical representation (USE THE SYMBOL g TO REPRESENT NUMBER OF km/L OF DIESEL FUEL THE TRUCK CAN GET)
2.4 Construct relevant hypotheses (THE RELATIONSHIP BETWEEN THE SPEED OF A TRUCK AND THE NUMBER OF km/L OF DIESEL FUEL THE TRUCK CAN GET)
2.5 Use mathematical knowledge appropriately (WRITE OUT THE FUNCTION OF TRANSPORTATION COSTS)

3. *Mathematical models → Mathematical solutions* (OPERATE MATHEMATICALLY)

3.1 Representational change (CONVERT THE RELATIONSHIP BETWEEN THE SPEED OF A TRUCK AND THE NUMBER OF km/L OF DIESEL FUEL THE TRUCK CAN GET INTO AN ALGEBRAIC EXPRESSION)
3.2 Analyse (VERIFY DIFFERENTIAL VARIABLES)
3.3 Apply the concept of derivatives (USE FIRST-ORDER DERIVATIVES TO SEEK EXTREMA)
3.4 Understand the meaning of parameters (MATHEMATICAL SOLUTIONS INCLUDE PARAMETERS)

4. *Mathematical solutions → Real-world meaning of solutions* (INTERPRET MATHEMATICAL RESULTS)

4.1 Verify mathematical results based on real-world situations (THE RATIONALITY OF f/w = 05)
4.2 Integrate arguments to verify interpretational results (THE RANGE OF f/w VALUE)

Figure 1: Framework showing transitions and mathematical modelling competencies in the implementation of transportation costs activity.

Then, students verified the variables and limitations in the situation to investigate the key factors influencing transportation costs. For instance, John suggested that the cost of diesel fuel is inversely proportional to transportation time. However, Mary had a different opinion, in that speed is not necessarily inversely proportional to the number of km/L of diesel fuel a truck uses. After group discussions, relevant factors were listed as below: the factors related to driving distance and the factors related to the truck. The preceding has demonstrated that all these students could

successfully generate a problem statement. Under considerations of the cost of diesel fuel and of the driver, the students needed to derive the optimal driving speed for the truck to minimise transportation costs.

Real-world models → Mathematical models. In the second stage, students engaged in the work of mathematisation. Students encountered the most difficulty and spent most of their time on this stage. The difficulty was in creating mathematical properties corresponding to situational conditions and hypotheses. It seemed rather important to provide these factors; for instance, *What are parameters?* and *What are variables?* This is also an important process in mathematical modelling activities.

All the students in Group D had the same hypotheses on the price per litre of diesel fuel, the hourly rate of a driver and the number of km a truck travels. However, their hypotheses on the speed of the truck and the number of km/L of diesel fuel a truck uses were slightly different. Someone hypothesised that a truck can run 40 km at a speed of 20 km/h, and that for every 20 km/h increase in speed, the number of km/L of diesel fuel a truck uses would be reduced by 10 km. Someone hypothesised that a truck can drive for one hour on one litre of diesel fuel at a speed of 50 km/h, and that an increase in speed of 5 km/h would reduce the driving time by 0.2 hours. Obviously, there was a significant difference between student performance and the meaning and purpose of mathematical modelling. Other groups of students had similar problems; thus, Sam re-explained the meaning of mathematical models and encouraged students to hypothesise more parameters and variables.

Group D first discussed whether several keywords in the problem were parameters or variables. Gray argued that since the number of km/L of diesel fuel a truck uses may vary with the speed of the truck, that this was a variable. Tom suggested that because the company does not often adjust salaries, the hourly rate for a truck driver can be considered as a parameter instead of a variable. Regarding the cost of diesel fuel, in John's opinion, even though the price of diesel fuel may be adjusted every week, the adjustments were relatively modest. Sometimes the price rises by two cents and then falls by two cents, meaning that the price does not change. Therefore, the price of diesel fuel can be hypothesised as a parameter. After a 20 minute discussion, Group D reached consensus that the heart of the question is the speed; any values directly related to the speed are variables and the rest are parameters. Therefore, all the students finished assigning symbols to represent all the parameters and variables.

The second stage of mathematisation was to construct hypotheses. A less controversial hypothesis was that a truck travels at a constant speed, ignoring traffic lights and jams. A more controversial hypothesis was the relationship between the speed of the truck and number of km/L of diesel fuel that the truck uses, which was the most important hypothesis in this modelling activity. When working on their own, most students in Group D hypothesised that the speed of a truck is inversely proportional to the number km/L of diesel fuel a truck uses. Such a hypothesis differed from real-world situations. Only Gray and Roberto put forward different opinions. Both of them suggested that the faster the speed, the more fuel that can be saved, based on their experiences in riding a scooter.

Hence, the students in Group D decided to investigate the data on their own regarding the speed of a truck and number of km/L of diesel fuel a truck uses, which could serve as a basis for the subsequent discussion. These students tried to identify the relationship between the speed of a truck and number of km/L of diesel fuel a truck uses by gathering relevant data from the Internet and dealer websites, and by calling car dealerships. The relationship between the speed of a truck and number of km/L of diesel fuel it uses was demonstrated by the students not to be a simple linear relationship.

For instance, John divided the speed of a truck into two different ranges, from 0 to 50 and from 50 to 80 km/h, and drew two line segments connecting two specific points (0,0) and (50,8), and (50,8) and (80,10), serving as the hypothesised graphical representations. After discussion, Group D hypothesised that the maximum speed was 100 km/h; beginning with 0 km/h, the number of km/L of diesel fuel a truck uses steadily increases with increases in speed; at a speed of 60 km/h, a truck can travel 12 km on 1 L of diesel fuel. They also hypothesised that after a truck goes over 60 km/h, the number of km/L of diesel fuel it uses steadily decreases; at a speed of 100 km/h, a truck can travel 8 km on 1 L of diesel fuel. Moreover, they represented their hypotheses with graphics.

After finishing its hypothesised graphical representations, Group D began with converting key factors into mathematical representations, including the travelling time (d/s), cost of a driver (wd/s), and cost of diesel fuel (fd/g), and created the function of transportation costs: C(x) = wd/s + fd/g. At this point, Group D finished its mathematical models, including the algebraic expression of the cost function and a graphical representation of the number of km/L of diesel fuel a truck uses.

Mathematical models → Mathematical solutions. In this stage, known mathematical knowledge was used to solve mathematical models. Because the form of cost functions differed from the functions with which students dealt previously, the first problem students encountered was that they did not know which variable should be differentiated. The next difficulty was how to use *g* to differentiate *s*. Students demonstrated the relationship between s and g using graphical representations; thus, they had difficulty implementing differential operations. After a five minute discussion, they were still unable to find a solution. Therefore, Sam suggested that the students convert graphics into functional forms by using the combination of an equation of two straight lines.

Finally, they used the concept of first-order derivatives to determine a mathematical solution. However, students wondered why the solution for the optimal speed contained parameters, because they thought that the form of an answer should be a number, based on their previous learning experiences. Sam discovered that most students had questions about the form of the mathematical solution. Hence, in class discussions, he explained the purpose and meaning of mathematical modelling and the meaning of a mathematical solution that included parameters.

Mathematical solutions → Real-world meaning of solutions. In this stage, students interpreted mathematical solutions as real results. In the last stage, Group D discovered that the optimal speed was correlated with the value of f/w. They also understood that they needed to propose possible results based on the value of f/w. Tom suggested that when f/w = 0.5, the optimal speed and lowest transportation costs can be obtained.

However, he did not consider the hourly rate of the driver to be unreasonable, which was only approximately 60 NTD when f/w = 0.5. Three students suggested that f/w should be 0.4 when the price per litre of diesel fuel was 29 or 30 NTD. Two students listed several solutions such as f/w = 0.2, *s* = 74.6; when f/w = 0.3, *s* = 65.9; when f/w = 0.4, s = 60.0; and when f/w = 0.5, then s = 55.6. They did not provide a single solution; instead, provided clients with the suggestion that the value of f/w be between 0.3 and 0.4.

CONCLUSIONS

This study emphasises student reflection on mathematical understanding, mathematisation and analysis of mathematical systems, and the interpretation of results to a given real-world problem, providing students with an opportunity to learn mathematics in a different way. This study replaced extreme problems in calculus courses with the mathematical modelling activities of reducing transportation costs. Through mathematical modelling instruction, students can gradually develop their mathematical modelling competency by working on their own and through discussion with their peers.

The results of research data analysis show that a fundamental and important problem encountered by students is their failure to recognise variables and parameters; and whether these values are known or unknown, obscure or clear, or independent or related. Without this fundamental knowledge, students may have difficulty engaging in mathematical modelling activities, especially during the process of mathematisation. Therefore, the insufficient ability of students to categorise variables and parameters should not be ignored. Educators should help students in establishing useful relationships required by mathematical problems. Another obvious problem is *representational change.*

Representational tools and systems, such as tables, graphics and drawings are important parts of the mathematical modelling process. This research result accords with the argument emphasised by Lesh and Doerr that representational fluency plays an important role in the model-documentation principle of mathematical modelling [2].

The study results verify the cognitive activities in which students engaged and the competency required during the mathematical modelling process of solving the problem of reducing transportation costs. Researchers and teachers can use the transitional framework proposed in this study to verify whether students have the specific abilities necessary to complete successfully particular mathematical modelling problems.

Teachers, who want to implement mathematical modelling instruction can also use the transitional framework to ensure that students are able to develop their mathematical modelling competency, even though not every modelling activity includes every element. From the perspective of student learning, the transitional framework can be used to predict difficulties that students may encounter.

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